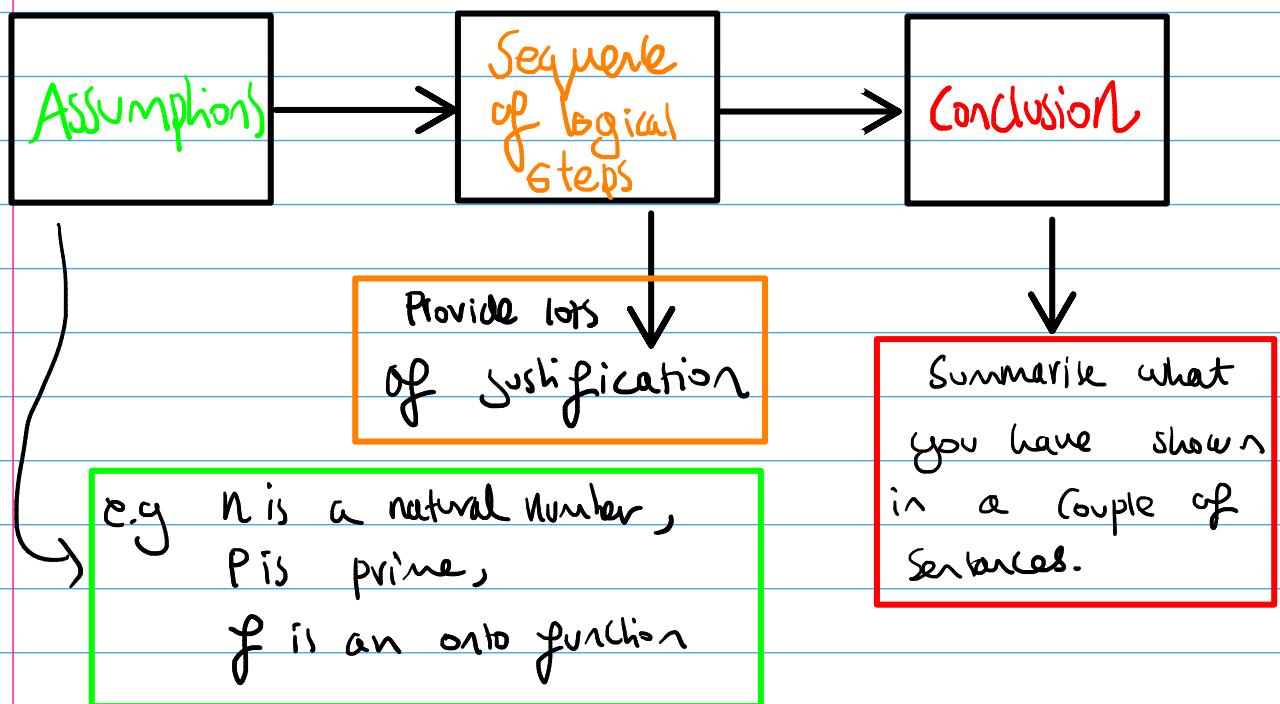


Proof



① Proof by deduction

① Start with assumptions

② Follow logical steps

③ Come to conclusion

Example from spec: Prove $n^2 - 6n + 10$ is positive for all $n \in \mathbb{N}$.

- ① Assume that n is a natural number
- ② $n^2 - 6n + 10 = (n-3)^2 + 1$ $(n-3)^2 \geq 0$
for all n . therefore $(n-3)^2 + 1 > 0$.
- ③ therefore, $n^2 - 6n + 10 > 0$ for all $n \in \mathbb{N}$

2) Proof by exhaustion

- ① start with assumption
- ② examine each separate case one by one
- ③ Come to final conclusion

Example:

Show $n^2 - 1$ is multiple of 3 if n is not.

① Assume that n is not a multiple of 3. that means n can be written in two ways

$3k+1$ or $3k+2$ for some $k \in \mathbb{N}$.

②

$$(a) \ n = 3k+1, \quad n^2 - 1 = (3k+1)^2 - 1 \\ = 9k^2 + 6k$$

So $n^2 - 1$ is a multiple of 3 $= 3(3k^2 + 2k)$

(b) $n = 3k+2$, then

$$n^2 - 1 = (3k+2)^2 - 1 = 9k^2 + 12k + 3 \\ = 3(3k^2 + 4k + 1)$$

So $n^2 - 1$ is a multiple of 3.

③ Since $n^2 - 1$ is a multiple of 3 for n of the form (a) and (b) the statement must be true.

3) disproof by counter example.

- ① Pick specific value such that statement is false
- ② Conclude the statement cannot be correct.

Example from spec:

Prove $n^2 - n + 1$ is not prime
for all natural numbers $n > 1$.

working
out.

Find an n such that $n^2 - n + 1$ is
not prime.

$$n=1 \quad n \neq 2 \quad n \neq 3 \quad n \neq 4 \quad \underline{\underline{n=5}}$$

Let $n=5$, then $n^2 - n + 1 = 5^2 - 5 + 1$
which is not prime - $\overset{= 21}{50}$ the
statement must be false.