# Proof

Assumptions → Sequence of logical steps → Conclusion

Provide lots of justification

Summarise what you have shown in a couple of sentences.

e.g $n$ is a natural number,
$P$ is prime,
$f$ is an onto function

## 1) Proof by deduction

1. Start with assumptions
2. Follow logical steps
3. Come to conclusion

Example from spec : Prove $n^2-6n+10$ is positive for all $n \in \mathbb{N}$.

① Assume that n is a natural number

② $n^2-6n+10=(n-3)^2+1$ $(n-3)^2 \geq 0$ for all n. therefore $(n-3)^2+1>0$.

③ therefore, $n^2-6n+10>0$ for all $n \in \mathbb{N}$

## 2) Proof by exhaustion

1. Start with assumptions
2. examine each seperate case one by one
3. Come to final conclusion

**Example:**

Show $n^2-1$ is multiple of 3 if $n$ is not.

1) Assume that $n$ is not a multiple of 3. that means $n$ can be can be written in two ways

$3k+1$ (a) or $3k+2$ (b) for some $k \in \mathbb{N}$.

2) (a) $n = 3k+1$, $n^2-1 = (3k+1)^2 - 1$

$= 9k^2 + 6k$

$= 3(3k^2+2)$

So $n^2-1$ is a multiple of 3

(b) $n = 3k+2$, then

$n^2-1 = (3k+2)^2 - 1 = 9k^2 + 12k + 3$

$= 3(3k^2 + 4k + 1)$

So $n^2-1$ is a multiple of 3.

3) Since $n^2-1$ is a multiple of 3 for $n$ of the form (a) and (b) the statement must be true.

## 3) disproof by counter example.

① Pick specific value such that statement is false

② Conclude the statement cannot be correct.

Example from spec:

Prove $n^2-n+1$ is not prime for all natural numbers $n>1$.

working out:

Find an $n$ such that $n^2-n+1$ is not prime.

$\cancel{n=1}$ $\cancel{n=2}$ $\cancel{n=3}$ $\cancel{n=4}$ $\underline{\underline{n=5}}$

Let $n=5$, then $n^2-n+1 = 5^2-5+1$

$= 21$

which is not prime. So the statement must be false.